## TD n° 2 - Simulation

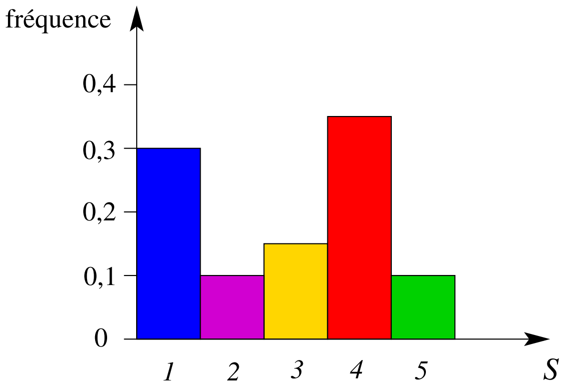
**Exercise 1.** *Alias method*

We are interested in the simulation of a discrete distribution on a *finite* set $S = \{1,\ldots,n\}$ (without loss of generality). Let $p_i$ the probability of occurrence of the value $i$, with $\sum_{i=1}^{n} p_i = 1$. You have at your disposal a function `Random` which returns a random real number in $[0,1]$ with uniform distribution and such that all calls are independent. We allow a pre-computation of time $O(n)$.

**1.** How to simulate this discrete distribution in time $O(\log n)$ and with one call to `Random`.

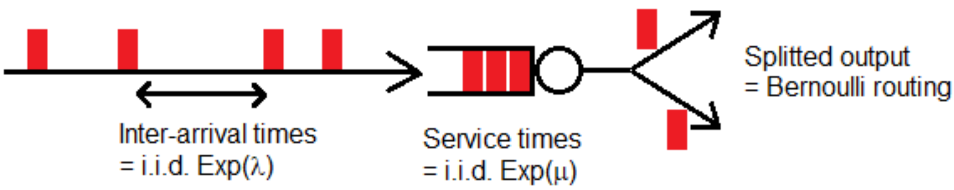
**2.** Design a method to simulate this discrete distribution in $O(1)$ time and calls to `Random`.

*Hint:* reshape the histogram of the distribution (where the bars indicate the frequency of each value).




**Exercise 2.** *Simulation of a communication channel*

Consider the following model of a communication channel in isolation (continuous time and discrete data):




- Traffic Input : packet arrivals follow a Poisson process of intensity $\lambda > 0$, that is let $T_n$ the arrival date of the $n$-th packet, then $T_0 = 0$ and inter-arrival $(T_n - T_{n-1})_{n\in\mathbb{N}^*}$ are i.i.d. of law $Exp(\lambda)$.
- Server: FIFO routing (First In, First Out) with exponential service of rate $\mu > 0$, that is let $S_n$ the time to process and send the $n$-th packet, then $(S_n)_{n\in\mathbb{N}^*}$ are i.i.d. of law $Exp(\mu)$.
- Queue: storage with $\infty$ memory.

**1.** Find a way to simulate the exponential law using the function `Random` of Exercise 1.

**2.** Use this simulation to write an algorithm simulating the model presented above.

**3.** Implement this simulation in python (see the attached skeleton example).

- Make a plot of the curve of the number of packets waiting in the queue in function of time.
- Let us consider a scenario when the output traffic is split, and the splitting follows Bernoulli law (with probability $p$ it goes to one direction and with probability $1-p$ it goes to another direction). Make the plots of the output traffics.
- Run some simulations, what can you say about the distribution of the interdeparture times (intervals of time between each departure) on each of the two output directions ?

You can then play with several different scenarios (for instance when `rate_arrival` < `rate_departure`, and when `rate_arrival` > `rate_departure`). What does your observation tell you?

**Exercise 3.** *Normal law*

Let $m, \sigma^2 \in \mathbb{R}_+$, a random variable $X$ follows a *normal law* $\mathcal{N}(m,\sigma^2)$ if $X$ takes its values in $\mathbb{R}$ and has density $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}}$.

**1.** Give an example of a phenomenon following a normal law.

**2.** Compute the mean and variance of $X$.

**3.** Let $X, Y$ be independent normal random variables of laws $\mathcal{N}(m_1,\sigma_1^2)$ and $\mathcal{N}(m_2,\sigma_2^2)$, show that $X+Y$ follows the law $\mathcal{N}(m_1+m_2,\sigma_1^2+\sigma_2^2)$.