Ch. 9.2 - Minimum Spanning Tree

Algorithm Design and Analysis [KOMS120403]
Lecture 11 (December 8, 2023)

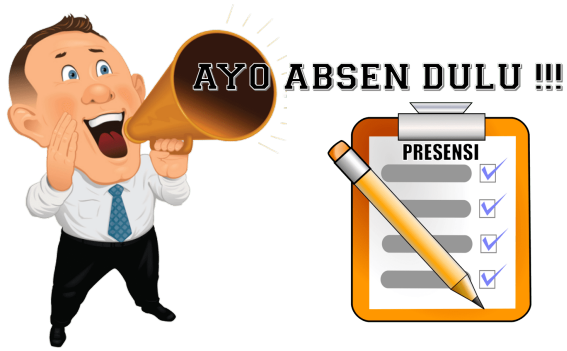
Dewi Sintiari

Computer Science Study Program
Universitas Pendidikan Ganesha
A.Y. 2022/2023

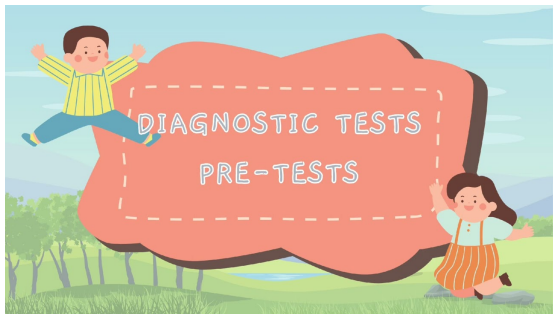
Good morning!

Embrace the challenge, study with passion.

- Have you filled the Attendance form on e-learning?



- Have you done the pretest on e-learning?



Dummy MK Desain dan Analisis Algoritma 2023

Topik/*Section* ini dibuat khusus untuk keperluan tes microteaching SKB CASN 2023.

Website Perkuliahan DAA Th. Akademik 2022/2023 — Mark as done

Ini adalah personal website saya yang memuat sebagian besar topik perkuliahan DAA. Silahkan mengunjungi webite tersebut.

Attendance — Mark as done

Perangkat pembelajaran — Mark as done

Silakan mecermati perangkat pembelajaran berikut agar Anda lebih siap untuk mengikuti perkuliahan.

1. Silabus
2. Kontrak Kuliah
3. Rencana Pembelajaran Semester
4. Rencana Tugas Mahasiswa

Buku referensi — Mark as done

Silakan unduh buku referensi pada tautan berikut.

- Anany Levitin - Introduction to the Design and Analysis of Algorithms-Pearson (2012)
- Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein - Introduction to Algorithms-MIT Press (2009)

Slide materi pertemuan ke-12 — Mark as done

Folder memuat dua materi, dalam Bahasa Indonesia dan dalam Bahasa Inggris. Anda memiliki kebebasan untuk membaca versi ID atau EN sesuai dengan kenyamanan Anda. Namun saya sangat menyarankan Anda untuk mempelajari versi EN untuk melatih literasi Bahasa Inggris Anda.

Tugas 5 (kelompok) — Mark as done

PETUNJUK

- Penamaan tugas menggunakan format "Tugas05_Kelompok-NomorKelompok" (**Contoh: Tugas05_Kelompok-7**).
- Perhatikan batas waktu pengumpulan, Anda tidak dapat mengumpulkan tugas setelah melewati tenggat waktu.
- Satu kelompok cukup mengumpulkan satu file, diwakilkan oleh salah seorang anggota kelompok.

Pre-test materi "Minimum Spanning Tree" — Mark as done

Post-test materi "Minimum Spanning Tree" — Mark as done

Review

- **Last week:** Greedy algorithm (for Knapsack problem, scheduling problem, etc.)
- **Next:** Greedy algorithm to solve problems in graph structure (namely, Minimum Spanning Tree problem)

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- Graph component (vertex, edge)
- Properties of graph (undirected, connected, acyclic)

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- **Last week:** Greedy algorithm (for Knapsack problem, scheduling problem, etc.)
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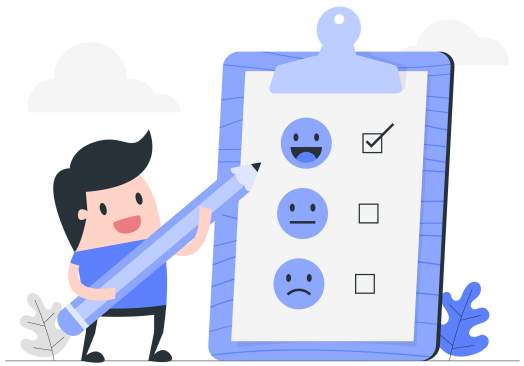
Graph:

- Graph component (vertex, edge)
- Properties of graph (undirected, connected, acyclic)
- Tree graph

Learning objective

After this lecture, you are expected to be able to:



1. Finding *Spanning Tree(s)* of a graph
2. Computing a *Minimum Spanning Tree* of a graph

Part 1.

Minimum Spanning Tree (MST)

Minimum Spanning Tree in real-life

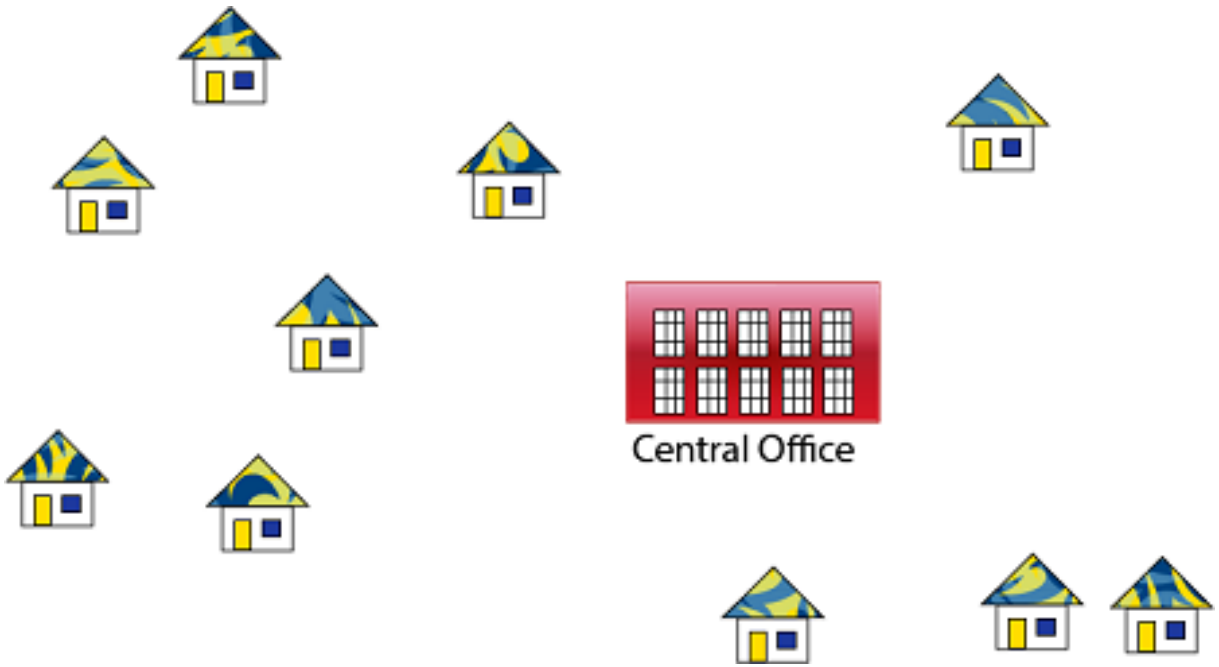
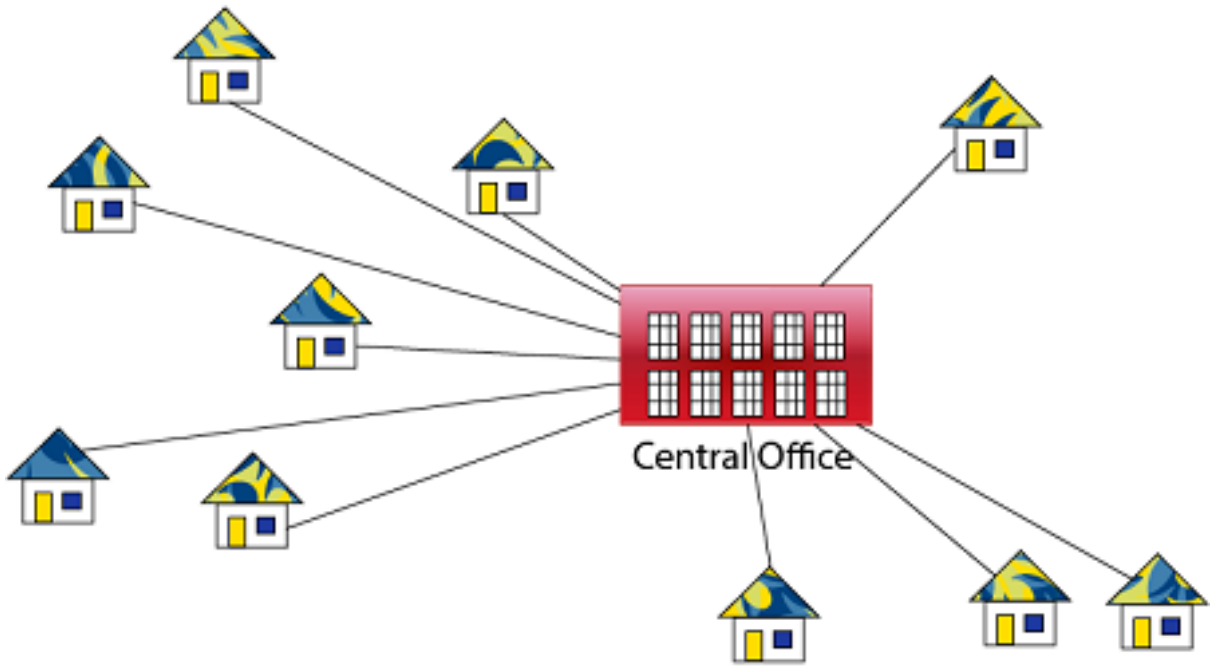


Figure: Building an electricity network *

*Image source:
https://www.javatpoint.com/applications-of-minimum-spanning-tree

Minimum Spanning Tree in real-life



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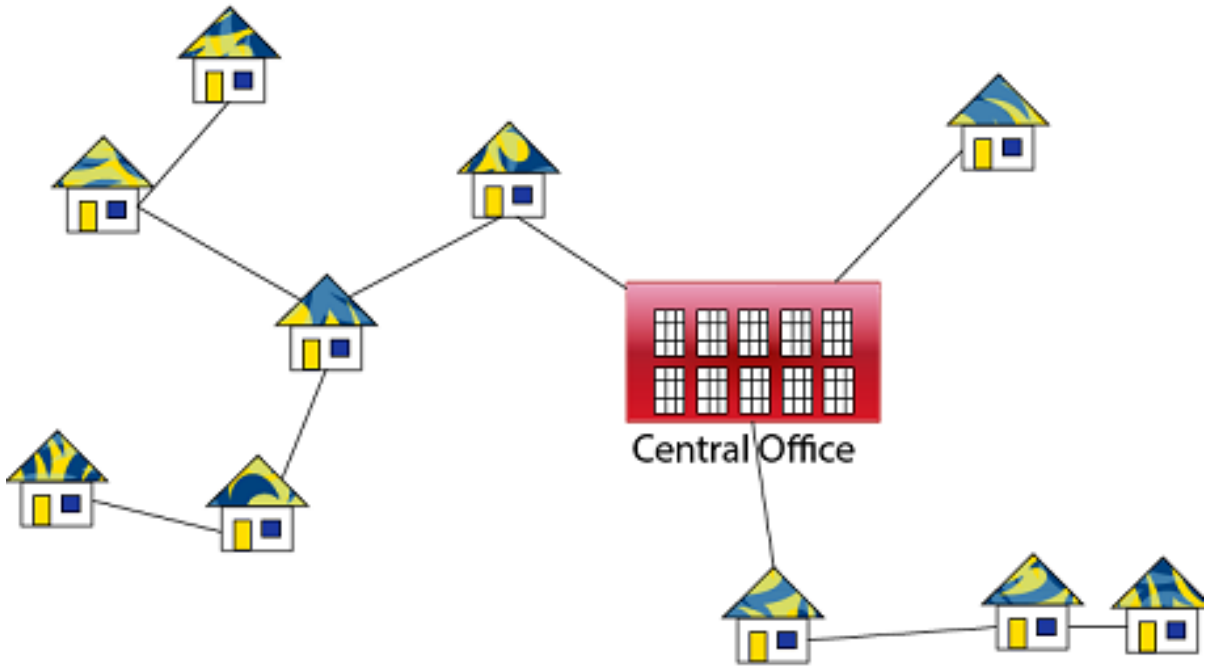

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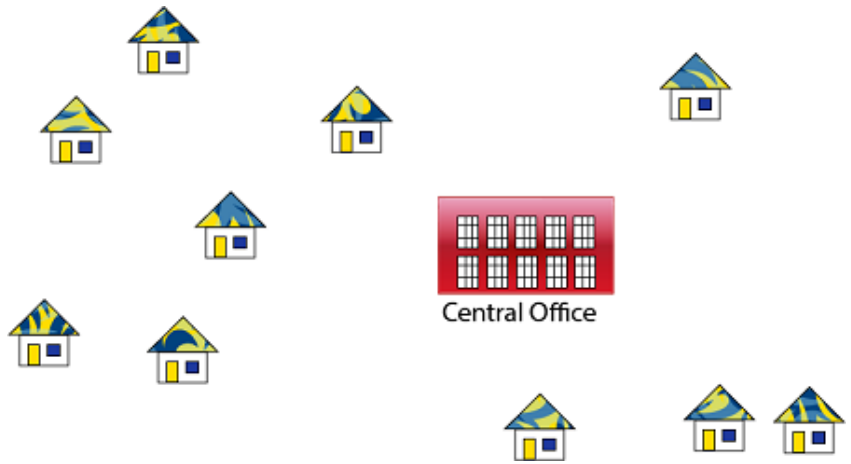

Goal: build a cable network at minimum cost

- Cables network = building connected graph that contains all vertices
- Minimum cost = it forms a tree;
  and the total weight of the edges is minimized.

The history of *Minimum Spanning Tree* problem *

- **Initial problem:** How to find the most economical power grid construction?
- Otakar Borüvka developed the first algorithm to find *MST*, in 1926.

Figure: Otakar Borüvka, Czech scientist (1899 - 1995)

MST application: network design (telephone, electrical, hydraulic, TV cable, computer, or road network in satellite), cluster analysis, real-time face verification.

*Source: `https://en.wikipedia.org/wiki/Minimum_spanning_tree`

Definition (*Spanning Tree*[a])

[a] Source: Book Analysis of Algorithms (Robert Sedgewick), Section 4.3

Given: A graph G, that is undirected and connected with weighted edges.

A *Spanning Tree* T is a subgraph of G that satisfies the following properties:

1. T is *spanning*, which means it contains all vertices;
2. T is a *tree graph*, which means it is connected and acyclic.

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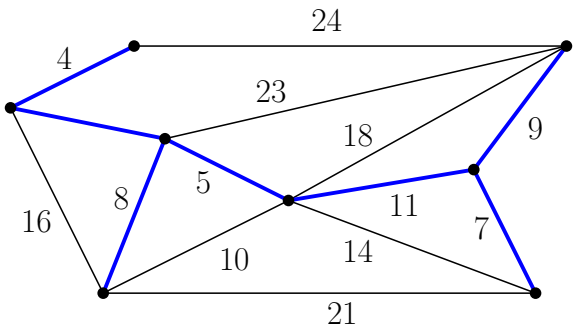
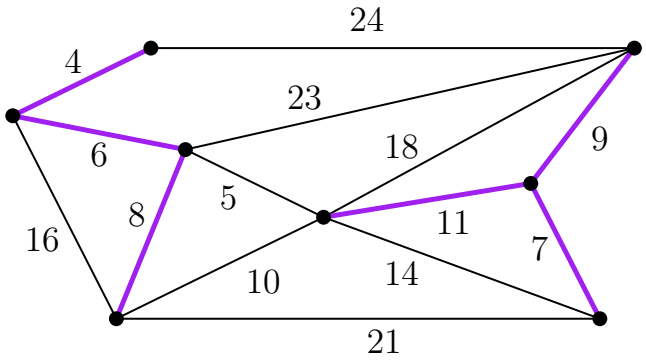
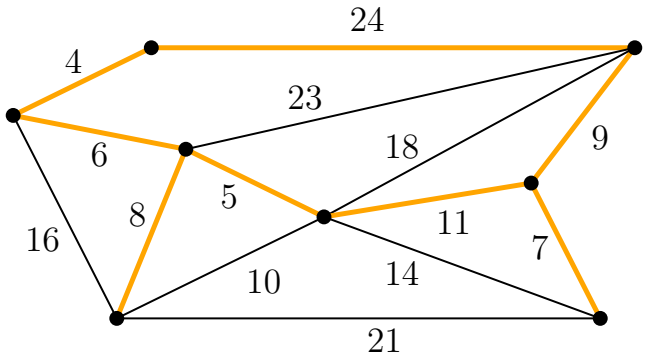


Figure: Example of *spanning tree* (colored blue) on a graph (colored black)

Q1: Which one is *Spanning Tree*?



spanning tree / not



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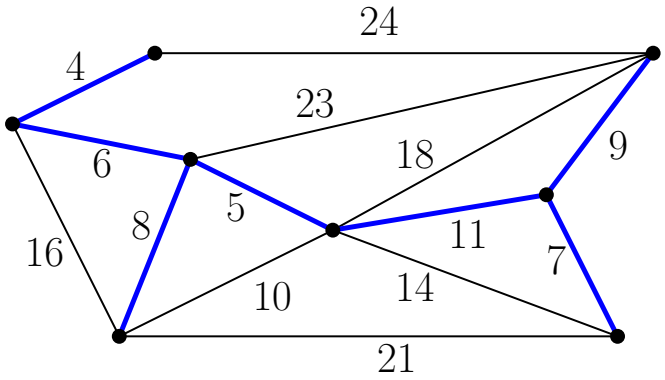

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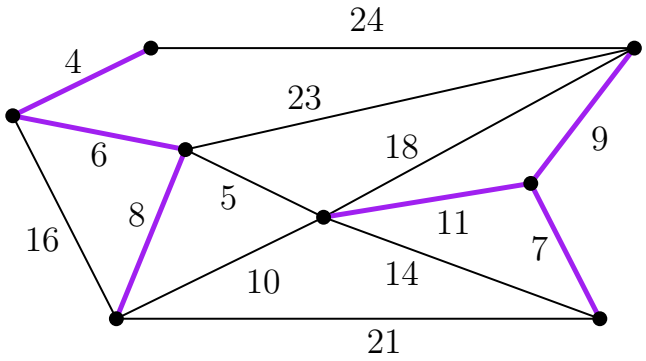
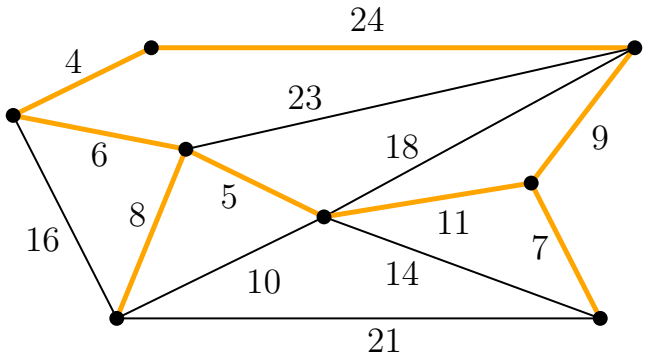
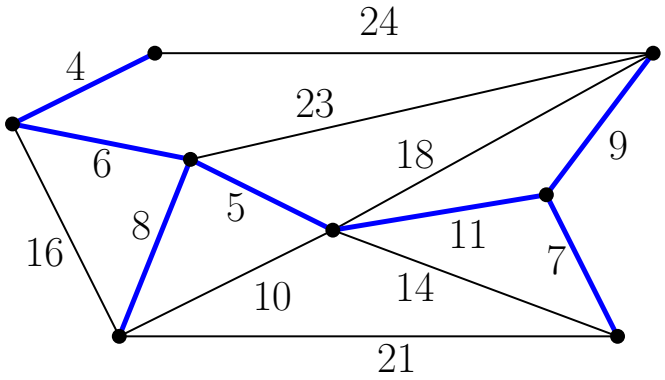


NO, because it is not connected



NO, because it is not acyclic



YES, *spanning tree*

Q2: Is it possible for a graph to have more than one *spanning tree*?

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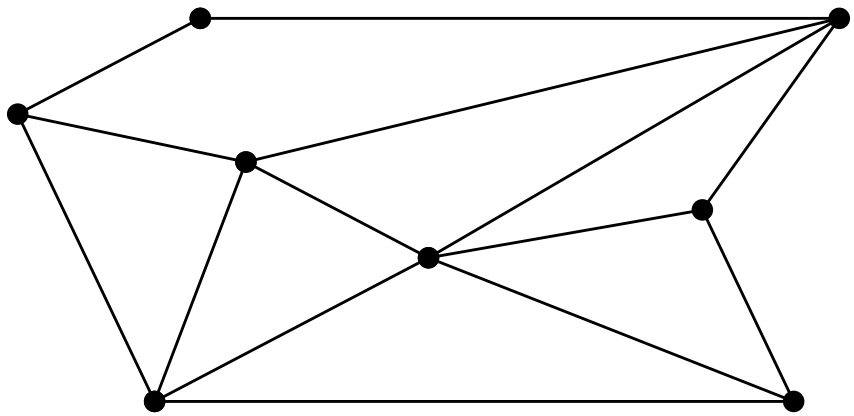

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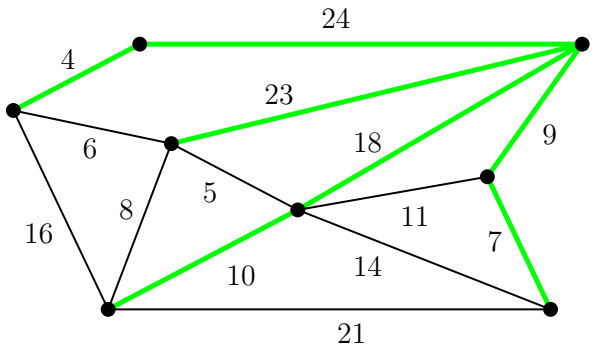
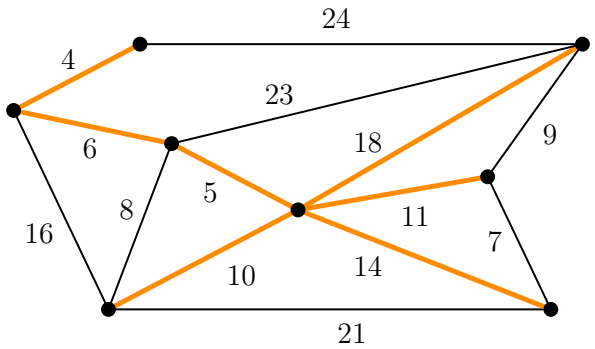
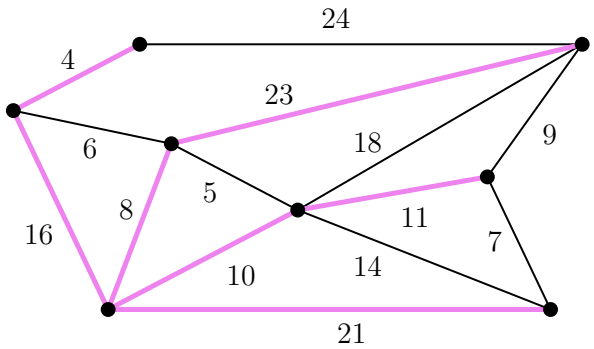


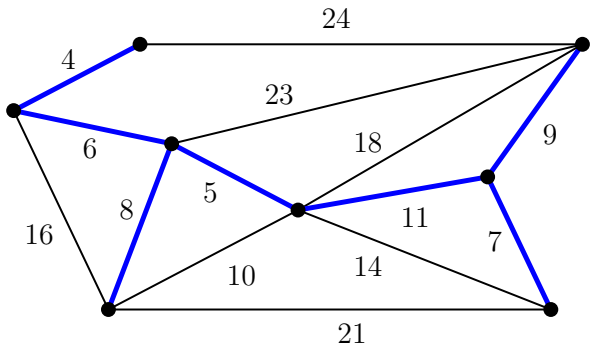
Weight $= 4 + 24 + 23 + 18 + 9 + 10 + 7 = 95$



Weight $= 4 + 6 + 5 + 10 + 18 + 11 + 14 = 68$



Weight $= 4 + 16 + 8 + 23 + 10 + 11 + 21 = 93$



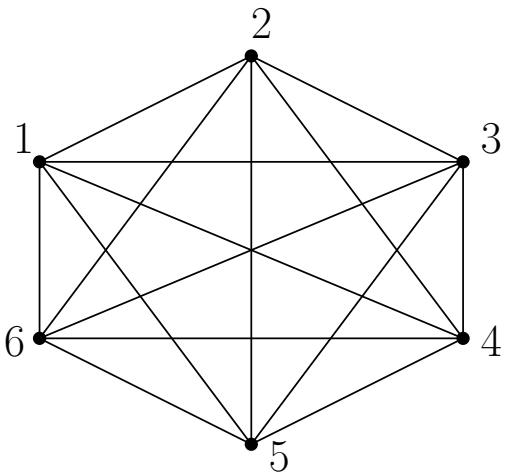
Weight $= 4 + 6 + 8 + 5 + 11 + 9 + 7 = 50$

The number of spanning trees in a graph

Remark

A graph can have more than one spanning tree, even can have so many

Example:



Complete graph with 6 vertices has

$6^{6-2} = 6^4 = 1296$ *spanning trees*

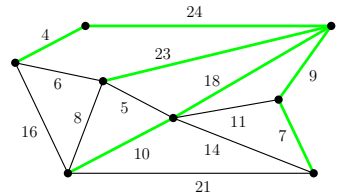
(by Cayley's Theorem, 1889)

Note: If there are so many spanning trees, then finding "Minimum Spanning Tree" is not so easy.

How to find *Minimum Spanning Tree* in a graph efficiently?

By brute-force algorithm:

1. List all spanning trees
2. Compute the weight of each spanning tree
3. Take the minimum weight spanning tree



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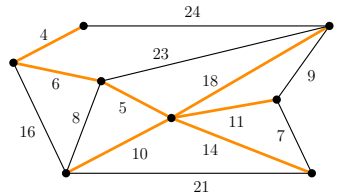
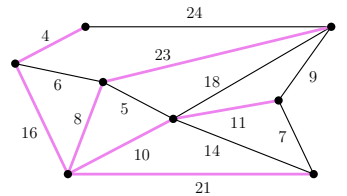


Weight = 4 + 6 + 5 + 10 + 18 + 11 + 14 = 68



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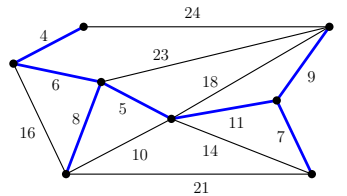

Weight = 4 + 6 + 8 + 5 + 11 + 9 + 7 = 50

Problem: Expensive cost (i.e., needs a big amount of time and resources)

Part 2. (*next week*)

Searching algorithm for MST

1. **Kruskal** algorithm

   (found by Joseph Kruskal (1956))

2. **Prim** algorithm

   (found by Vojtěch Jarník (1930), dan Robert C. Prim (1957))

†

† **Note:** There are other algorithms, but they are not discussed in this lecture

Conclusion

- Spanning tree is:
- The characteristics of Minimum Spanning Tree is:
- Two searching algorithms solving Minimum Spanning Tree problem:

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  - it forms a tree;
  - it has the minimum total weight.
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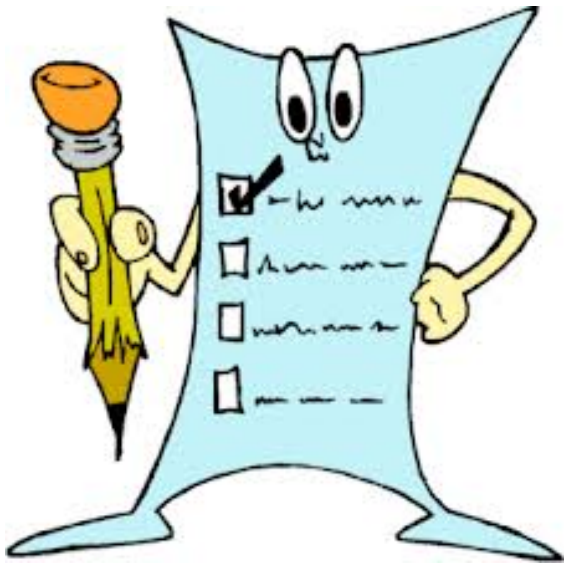
  - it contains all vertices;
  - it is connected;
  - it forms a tree;
  - it has the minimum total weight.
- Two searching algorithms solving Minimum Spanning Tree problem: *Kruskal Algorithm & Prim Algorithm*

References

1. **Book:** *Introduction to Algorithms* - T.H. Cormen, C.E. Leiserson, R.L. Rivest, C. Stein (2001)
2. **Book:** *Introduction to the Design and Analysis of Algorithms* - Anany Levitin (2012)
3. **Book:** *An Introduction to the Analysis of Algorithms* - Robert Sedgewick (2012)
4. **Journal paper:** *Minimum-weight spanning tree algorithms: A survey and empirical study* - Cüneyt F. Bazlamaçci, Khalil S. Hindi (1999)
5. **Youtube video:** *Kruskal's algorithm in 2 minutes*, `https://www.youtube.com/watch?v=71UQH7Pr9kU`
6. **Youtube video:** *Prim's algorithm in 2 minutes*, `https://www.youtube.com/watch?v=cplfcGZmX7I`

Time for post-test



Please access your e-learning page. $\longrightarrow$

Assignment (*case method / case-based exploration*)

Tugas 5 (kelompok)

Mark as done

Opens: Friday, 8 December 2023, 12:00 AM
Due: Thursday, 14 December 2023, 10:00 PM

PETUNJUK

- Penamaan tugas menggunakan format "Tugas05_Kelompok-NomorKelompok" (**Contoh: Tugas05_Kelompok-7**).
- Perhatikan batas waktu pengumpulan, Anda tidak dapat mengumpulkan tugas setelah melewati tenggat waktu.
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LKM.pdf	4 December 2023, 6:09 PM
Materi 11.1 - Kruskal.pdf	4 December 2023, 2:59 PM
Materi 11.2 - Prim.pdf	4 December 2023, 2:59 PM

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thank you, & good luck...!