Linear Algebra

[KOMS120301] - 2023/2024

14.1 - Intuition behind eigenvectors

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Learning objectives

- Recap what we learned in the previous weeks;
- Get an intuitive understanding of the concept;
- Relate it to the concept of linear transformation.

What we have learned

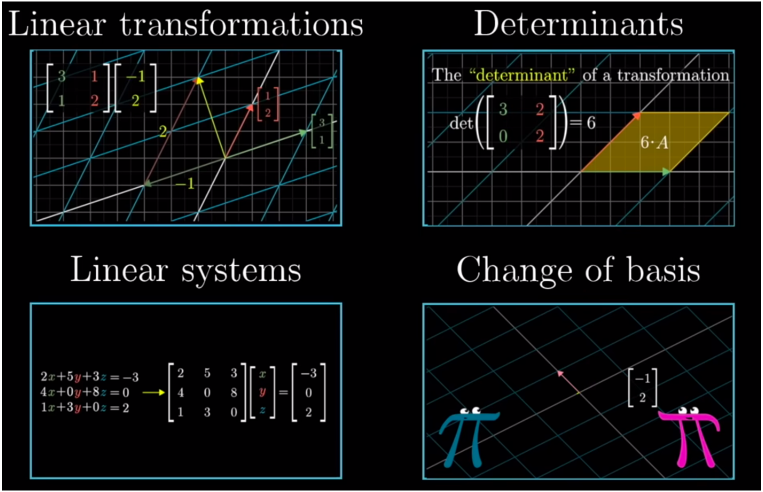
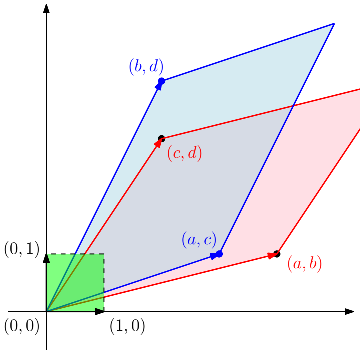


Figure: Prerequisites (*source: Youtube of 3Blue1Brown*)

Geometric interpretation of **determinant** (*from Week 5*)



Matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ can be viewed as an "arrangement" of:

- row vectors: $\begin{bmatrix} a & b \end{bmatrix}$ and $\begin{bmatrix} c & d \end{bmatrix}$
- or, column vectors: $\begin{bmatrix} a \\ c \end{bmatrix}$ and $\begin{bmatrix} b \\ d \end{bmatrix}$

The matrix defines the so-called *linear transformation* of the unit square (in green) formed by the *basis vectors* $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, with respect to:

- the row vectors, shown by the red parallelogram; or
- the column vectors, shown by the blue parallelogram

Both parallelograms have the **same area**. Prove it!

Vectors that "stay in their position" after transformation

Transformation of basis vectors (1)

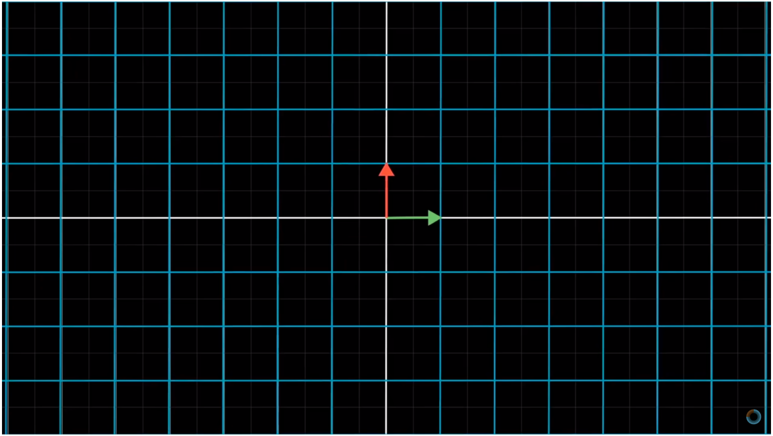


Figure: Two basis vectors in standard system (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (2)

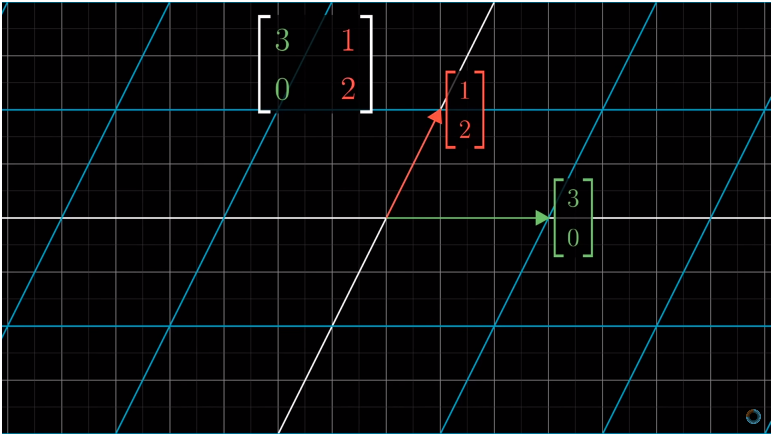


Figure: Result of transformation of the basis vectors remain in its "position" (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (3)

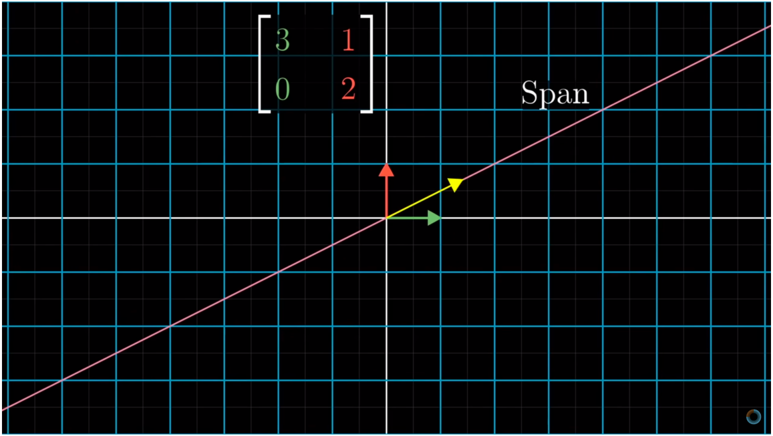


Figure: A (yellow) vector and its span (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (4)

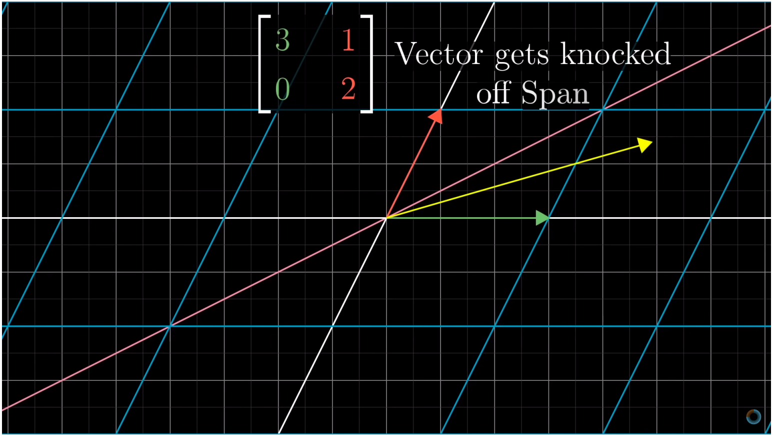


Figure: The yellow vector does not stay in its position (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (5)

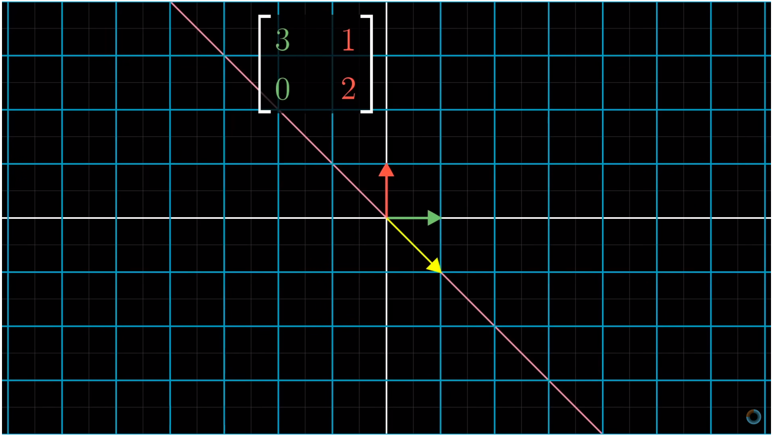


Figure: Another yellow vector (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (6)

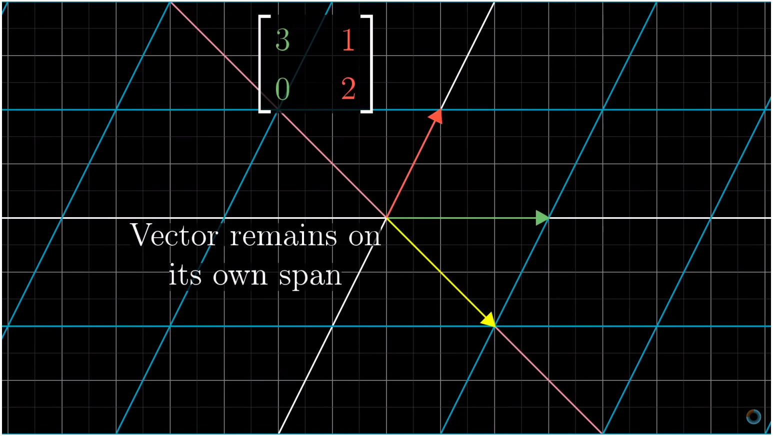


Figure: The vector remains in its position after transformation (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (7)

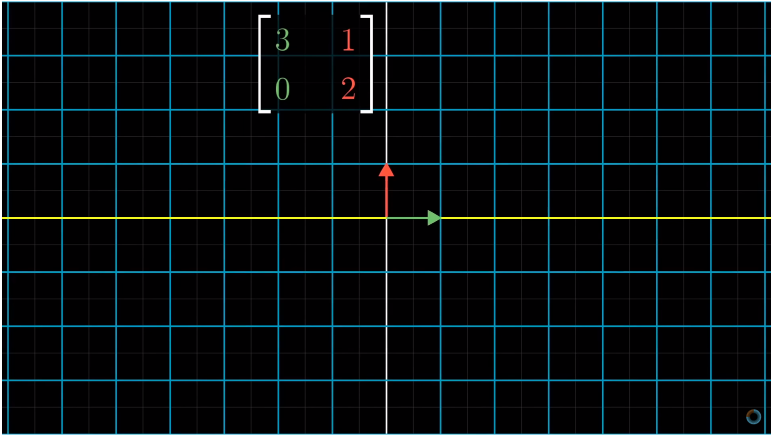


Figure: What happens to the green basis vector and its span? (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (8)

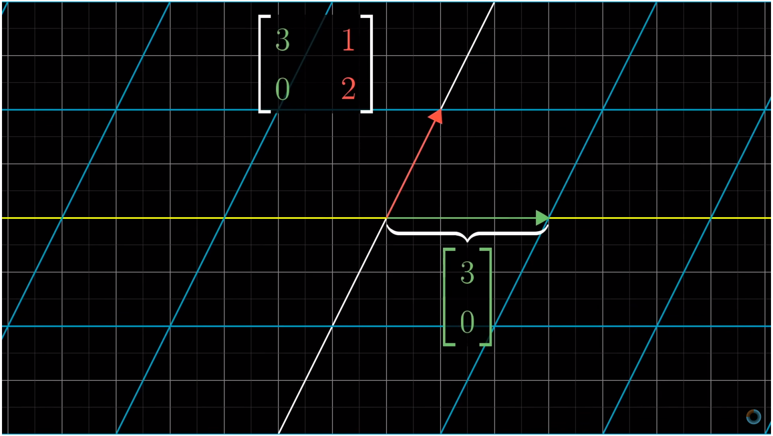


Figure: The green vector remains in its position, and multiplies by 3 (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (9)

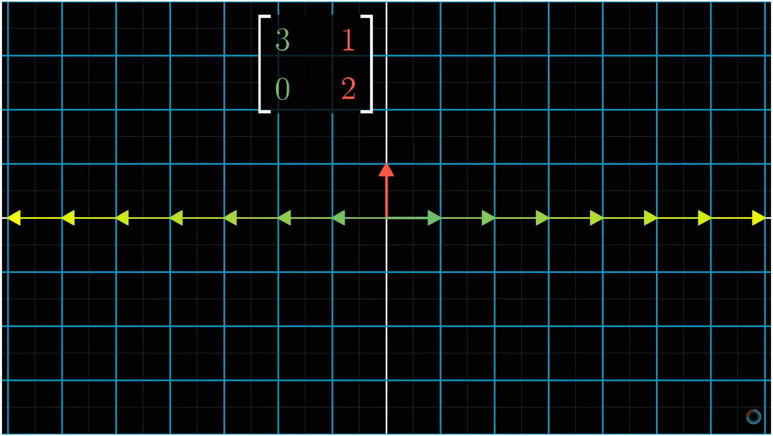


Figure: This happens to all vectors with the same (reverse) direction as the green vector (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (10)

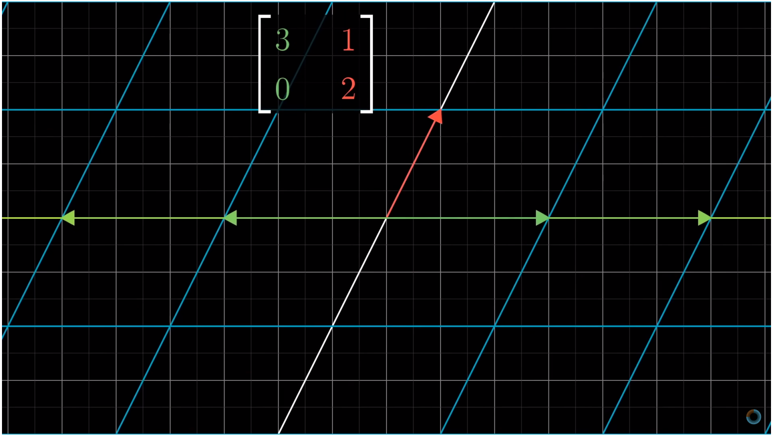


Figure: They are all stretched to 3 times the original vector (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (11)

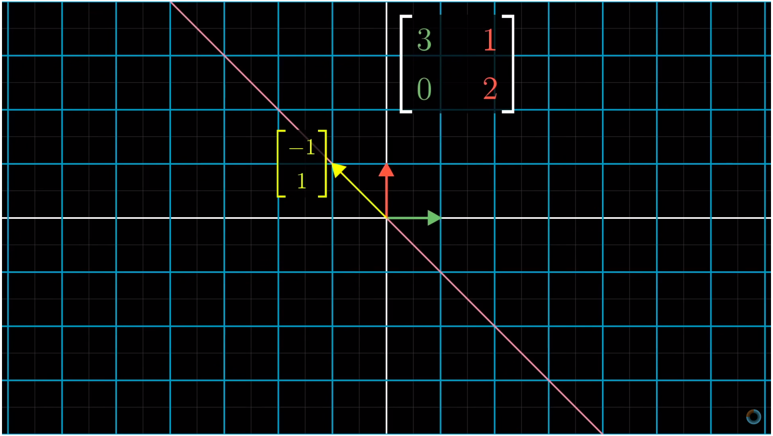


Figure: Another vector with similar property (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (12)

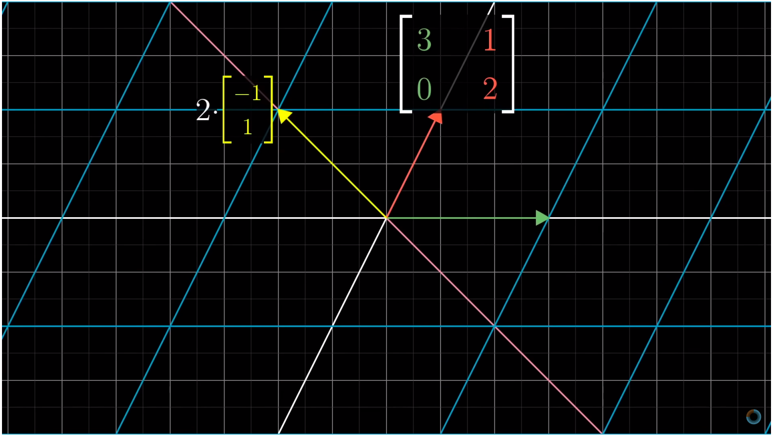


Figure: This vector remains in its position after transformation (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (13)

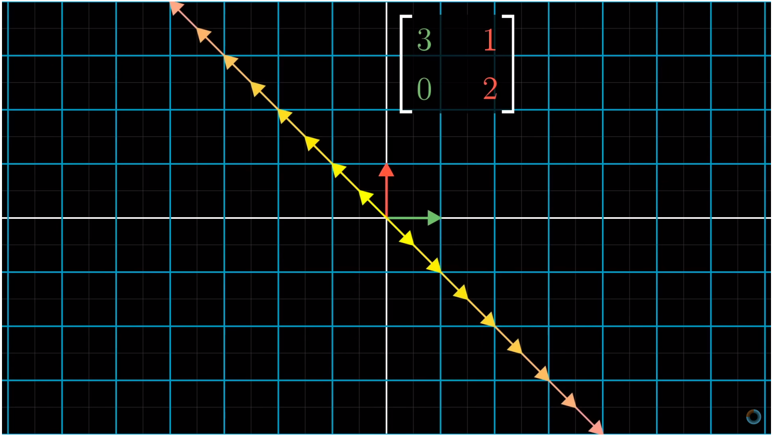


Figure: The property holds for all vectors in the span of its vector (*source: Youtube of 3Blue1Brown*)

Eigenvectors (1)

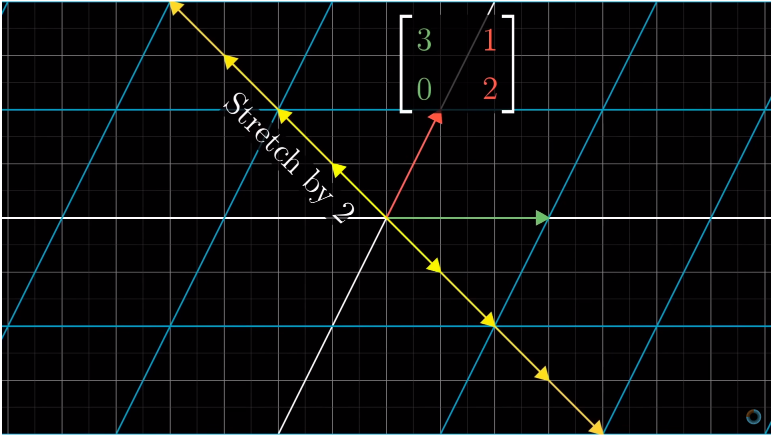


Figure: The yellow vector is stretched by 2 (*source: Youtube of 3Blue1Brown*)

Eigenvectors (2)

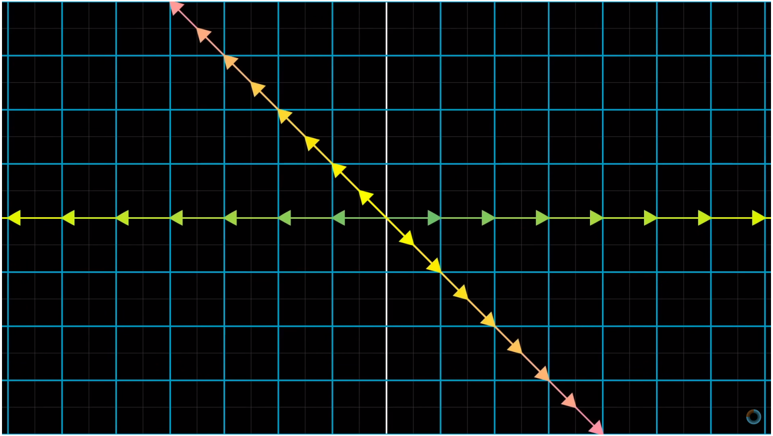


Figure: The green vector is stretched by 3 (*source: Youtube of 3Blue1Brown*)

Eigenvectors (3)

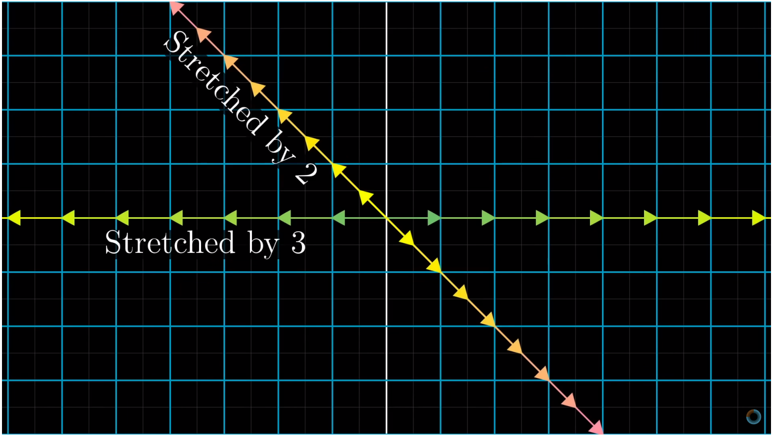


Figure: *Source: Youtube of 3Blue1Brown*

Eigenvectors (4)

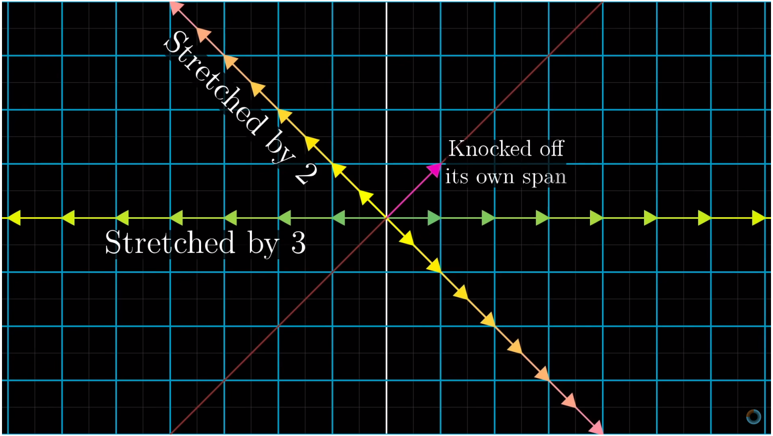


Figure: Other vectors do not stay in their span *Source: Youtube of 3Blue1Brown*

Eigenvectors (5)

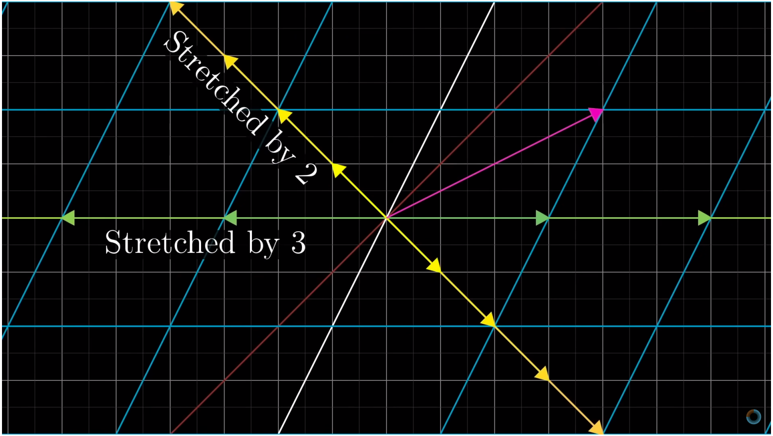


Figure: The transformation keeps the two vectors (yellow and green) in their position (*source: Youtube of 3Blue1Brown*)

Eigenvectors (6)

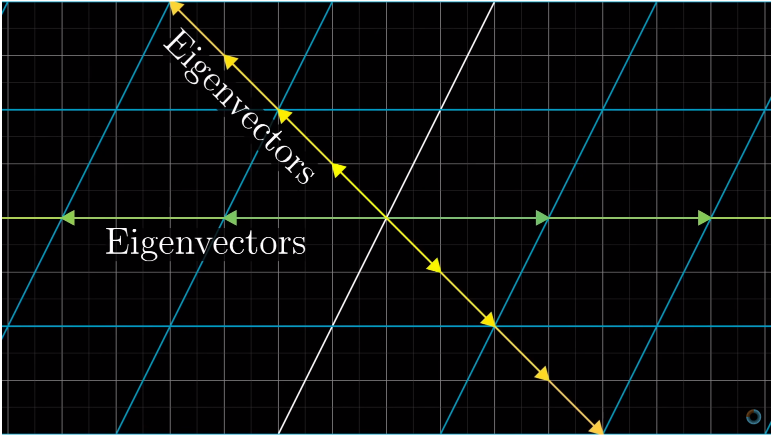


Figure: The transformation keeps the two vectors in their position (*source: Youtube of 3Blue1Brown*)

Eigenvectors (7)

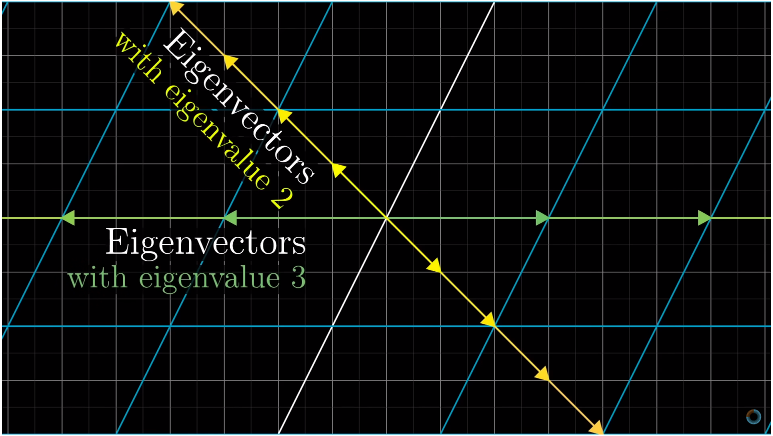


Figure: They are called eigenvectors (The transformation keeps the two vectors in their position *source: Youtube of 3Blue1Brown*)

Eigenvectors (8)

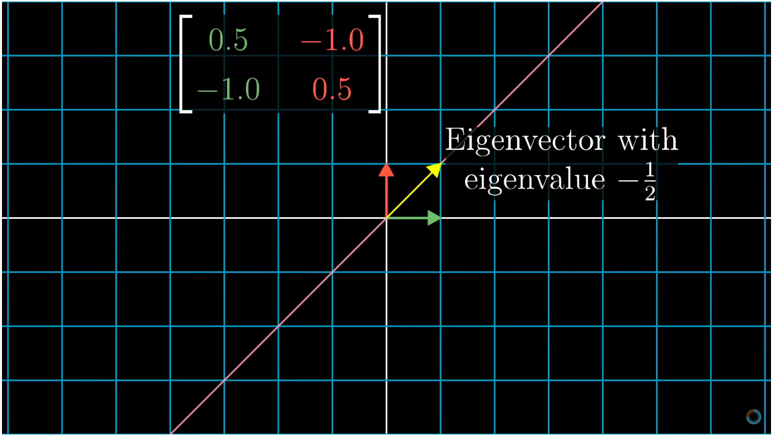


Figure: Prerequisites (*source: Youtube of 3Blue1Brown*)

Eigenvectors (9)

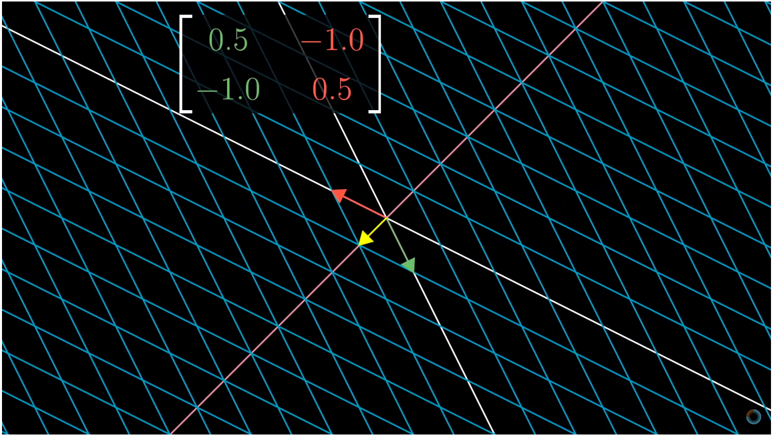


Figure: Prerequisites (*source: Youtube of 3Blue1Brown*)