Linear Algebra

[KOMS120301] - 2023/2024

4.3 - Applications of Linear System in CS

(the content of this slide is adapted from the lecture's slide of Rinaldi Munir, ITB)

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Learning objectives

After this lecture, you should be able to:

1. explain an application of linear system, especially in the polynomial interpolation.

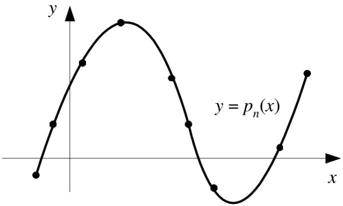
Polynomial interpolation

Problem

Given $n+1$ points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$. Determine polynomial $p_n(x)$ that goes through the points, s.t.,

$$y_i = p_n(x_i) \quad \textit{for } i = 0, 1, 2, \ldots, n$$

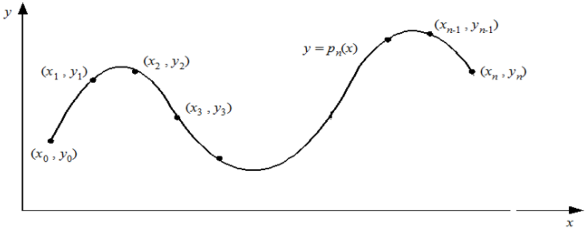
After the polynomial $p_n(x)$ is found, $p_n(x)$ can be used to compute the estimation of the y-value in $x = a$, that is $y = p_n(a)$.



Polynomial interpolation

The polynomial interpolation of degree n that pass through points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ is:

$$p_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

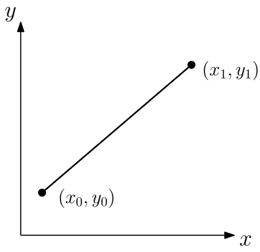


Linear interpolation

Linear interpolation is an interpolation of two points with a linear line.

Let given two points (x_0, y_0) and (x_1, y_1). Polynomial that interpolate the two points is:

$$p_1(x) = a_0 + a_1 x$$



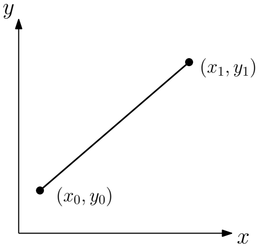
$$y_0 = a_0 + a_1 x_0$$
$$y_1 = a_0 + a_1 x_1$$

This can be solved using Gaussian elimination.

Quadratic interpolation

Let given three points (x_0, y_0), (x_1, y_1), and (x_2, y_2). Polynomial that interpolate the three points is:

$$p_1(x) = a_0 + a_1 x + a_2 x^2$$



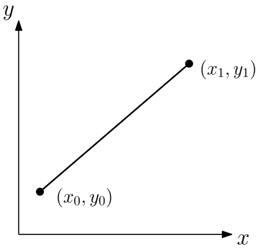
$$y_0 = a_0 + a_1 x_0 + a_2 x_0^2$$
$$y_1 = a_0 + a_1 x_1 + a_2 x_1^2$$
$$y_2 = a_0 + a_1 x_2 + a_2 x_2^2$$

This can be solved using Gaussian elimination.

Cubic interpolation

Let given four points (x_0, y_0), (x_1, y_1), (x_2, y_2), and (x_3, y_3).
Polynomial that interpolate the four points is:

$$p_1(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$



$$y_0 = a_0 + a_1 x_0 + a_2 x_0^2 + a_2 x_0^3$$
$$y_1 = a_0 + a_1 x_1 + a_2 x_1^2 + a_2 x_1^3$$
$$y_2 = a_0 + a_1 x_2 + a_2 x_2^2 + a_2 x_2^3$$
$$y_3 = a_0 + a_1 x_3 + a_2 x_3^2 + a_2 x_3^3$$

This can be solved using Gaussian elimination.

General interpolation

Similarly, using the Gaussian elimination method, we can interpolate polynomial of degree n for $n \geq 4$, given $(n+1)$ data.

$$
\begin{aligned}
y_0 &= a_0 + a_1 x_0 + a_2 x_0^2 + \cdots + a_n x_0^n \\
y_1 &= a_0 + a_1 x_1 + a_2 x_1^2 + \cdots + a_n x_1^n \\
y_2 &= a_0 + a_1 x_2 + a_2 x_2^2 + \cdots + a_n x_2^n \\
&\vdots \qquad\qquad \vdots \\
y_3 &= a_0 + a_1 x_3 + a_2 x_3^2 + \cdots + a_n x_n^n
\end{aligned}
$$