TD n°11 (Poisson Process)

Exercice 1. *Addition of two independent Poisson process*

Let $(N_t^X)_t$ and $(N_t^Y)_t$ be two independent Poisson process, of intensity λ and μ. Let $(N_t^Z)_t$ be the process obtained by the addition of $(N_t^X)_t$ and $(N_t^Y)_t$. Prove that $(N_t^Z)_t$ is a Poisson process and find its intensity.

Exercice 2. *So, a fish walks into a bar...*

Clients enter into a bar following a Poisson process of intensity λ. Knowing that two clients entered during the first hour, what is the probability that :

1. both clients entered during the first 20 minutes?

2. at least one client entered during the first 20 minutes?

Exercice 3. *The bus paradox*

A bus company assures that the arrival time of the buses to a bus stop is modeled by a Poisson process $(X_n)_n$ of intensity $\lambda = 0.1$ *bus per minute* and, thus, the average time between two buses is 10 minutes.
A client, Mister Relou, takes the bus every day at time $t \in \mathbb{R}$ after the beginning of the service and notes every day his waiting time at the bus stop. Mister Relou considers that he should wait in average 5 minutes *"because"* the average time between two buses is 10 minutes. He notices that he is waiting 10 minutes in average and complains to the bus company.

1. Calculate the probability that Mister Relou misses the n-th bus but catches the $(n+1)$-th bus.

2. Calculate the probability that he misses the n-th bus and has to wait the $(n+1)$-th bus for at least s minutes.

3. Calculate the probability that he has to wait the next bus for at least s minutes.

4. Deduce the average waiting time and answer to Mister Relou.

Exercice 4. *Two Poisson process*

A database can have two types of request : writing and reading. They happen following two independent Poisson process of parameters λ_R and λ_W.

1. What is the probability that the interval between two reading is bigger than t?

2. What is the probability that the next event is a reading?

3. What is the probability that at most n writings happen in the interval $[0, t[$?

4. What is the probability that at least two events (reading or writting) happen in the interval $[0, t[$?

Exercice 5. *Traffic Lights*

On an avenue in Monaco, the car traffic is modeled by a Poisson process of intensity $\lambda = 2$ cars per minute.

1. Use a famous theorem to give a Gaussian/Normal approximation of the law of T_n, the arrival time of the n-th car.

Because of a traffic light, the flow of cars is regularly stopped for t minutes. We assume that when the traffic light turns green, all the waiting cars can go through the intersection. We also assume that all the cars in Monaco are limousines that occupy 10 meters in average while stopped.

2. In order to set t, how long can we stop the traffic if we want that the resulting queue exceeds a length of 250 meters only with a probability 0.2? (*Reminder : the value of x for which $\mathbb{P}\{\mathcal{N}(0,1) < x\} = 0.2$ is* $x \approx -0.85$)