# HW 3: Channel coding

(due October 25th, before tutorial)

1. Consider a source given by $X^n = X_1 \dots X_n$ with $X_i$ independent and identically distributed bits with $\mathbf{P}\{X_i = 1\} = \frac{1}{4}$. Describe the distribution of the random variable $h_{X^n}(X^n) = -\log_2 P_{X^n}(X^n)$. How many values does it take? What is the probability for each different value? What is the expectation?

2. Consider the channel $W$ with input alphabet $\mathcal{X} = \{a, b, c\}$ and output alphabet $\{0, 1\}$, with $W(0|a) = 1, W(0|b) = \frac{1}{2}, W(1|b) = \frac{1}{2}$ and $W(1|c) = 1$. Then, let $W^{\times n}$ be $n$ independent copies of $W$.

   (a) For any $M$, determine the optimal (i.e., smallest possible) error probability for an $M$-code for $W^{\times n}$, as a function of $M$ and $n$.

   (b) Compute $\mathrm{C}(W)$.

3. Let $a \in \{1, 2\}$. Consider the additive noise channel with input alphabet $\mathcal{X} = \{0, 1\}$ and output alphabet $\mathcal{Y} = \{0, 1, 2, 3\}$, where the output $Y$ is given by $x + Z$ when $x$ is the input symbol and $Z$ is a random variable with distribution $\mathbf{P}\{Z = 0\} = \mathbf{P}\{Z = a\} = \frac{1}{2}$. Compute the information capacity of this channel.